

Evidence of nonlocality due to a gradient term in the optical model

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Abstract

We demonstrate that the presence of a velocity-dependent term in the phenomenological optical potential simulates a source of nonlocality. This is achieved by showing that, in the interior of the nucleus, the nonlocal wave functions are different from the corresponding local ones obtained in the absence of the velocity-dependent term in accordance with the Perey effect. It is also shown that the enhancement or suppression of the nonlocal wave function is energy as well as angular momentum dependent. The latter is in line with the results of previous works that introduced parity dependent terms in the conventional optical potential.

Keywords: Damping factor, Perey effect, Optical potential.

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1 Introduction

It is a well known fact that the nonrelativistic nucleon-nucleus optical potential is nonlocal and energy dependent [1, 2]. Nevertheless, a common approach to studying the elastic NA scattering is the use of phenomenological local optical models, and a variation of the model parameters with the incident projectile energy can be taken as a sign of the presence of nonlocal effects in the scattering process [3]. For example, a recent work presented local and global optical potentials for neutron and proton elastic scattering from nuclei that fall in the mass range $24 \geq A \geq 209$ [4], while nucleon incident energies ranged from 1 keV to 200 MeV. The authors obtained excellent elastic angular distribution fits using local optical potentials each consisting of 20 fit parameters, however, the strength of the central real part showed the largest variation with incident energy. Another measure of nonlocality is given by the size of the Perey damping factor, as will be discussed in the present study for the case of the presence of a gradient term in the optical potential. Such a gradient term, also denoted as velocity dependent term, is usually absent in conventional optical model potentials, but has been introduced in a previous study [5], and will be the object of further analysis in the present investigation.

The nonrelativistic optical potential contains several sources of nonlocalities. One such nonlocality is due to the Pauli exclusion principle and can be taken into account by antisymmetrizing the wave function as in the Hartree-Fock theory [6]. Further, in the Hartree-Fock equation, the nonlocality due to exchange effects can be expressed in terms of a spatially variable mass [7]. More recently, the importance of accounting for the Pauli principle at incident energies greater than 25 MeV has been established [8]. At low energies the importance of the Pauli exclusion principle has been investigated in collective-model coupled-channel calculations [9]. A second source of nonlocality is due to the coupling of the inelastic excitations to the nuclear ground state during the scattering process [2], that gives rise to a nonlocal dynamic polarization potential in the elastic channel. Since this nonlocality is difficult to take into account rigorously, it is usually accounted for by numerically coupling a few important inelastic channels to the elastic channel [10, 11]. However, when the energy of the incident nucleon is low enough, the scattering process is strongly affected by the individual discrete states of the target nucleus. A recent study that employed the multichannel algebraic scattering theory (MCAS) to explore the dynamic polarization potential [3, 12] for low

incident energies also showed that the resulting optical potential was strongly nonlocal and energy dependent.

Recently, we introduced a novel gradient term in the Schrödinger equation describing the NA elastic scattering process. We interpreted this term as a change in mass of the incident nucleon when moving inside the nuclear matter of the target nucleus [5]. The mass change could be a consequence of the nucleon-nucleon interaction being affected by the presence of surrounding nucleons, like a surface tension which is present only at the surface of the target nucleus. This view is supported by the fact that our model, in Ref. [5], resulted in excellent fits to the $N - ^{12}C$ elastic scattering data when the coefficient of the derivative term was assumed to be proportional to the gradient of the nuclear density, which is most important at the nuclear surface. Furthermore, our model reproduced well the large angle scattering minima that are usually associated with nonlocalities [13, 14]. However, it is still not yet clear whether the gradient term also simulates other sources of nonlocal effects in the phenomenological optical model like, for examples, the Pauli exclusion principle and coupling of the elastic channel to the inelastic ones.

We formulated the mass dependence by introducing a Kisslinger type potential into the conventional optical model [15]. This is achieved by writing the Schrödinger operator as

$$-\nabla \cdot \frac{\hbar^2}{2m^*(r)} \nabla + V - E = -\frac{\hbar^2}{2m_0} \nabla^2 + \hat{V}(r, p) - E, \quad (1)$$

where the resulting potential \hat{V} is velocity-dependent and has the form:

$$\begin{aligned} \hat{V}(r, p) &= V(r) + \frac{\hbar^2}{2m_0} \nabla \cdot \rho(r) \nabla \\ &= V(r) + \frac{\hbar^2}{2m_0} [\rho(r) \nabla^2 + \nabla \rho(r) \cdot \nabla]. \end{aligned} \quad (2)$$

The velocity-dependent potential $\hat{V}(r, p)$ plays the role of a nonlocal potential $V(r, r')$. In fact there is an equivalence between a velocity-dependent (or momentum-dependent) potential and a nonlocal potential $V(r, r')$ [1]. This can be seen by interpreting the gradient term that acts on the wave function as the first term of a Taylor series that displaces the wave function from point \vec{r} to a different location. Furthermore, the spatially variable effective mass

$m^*(r)$ is assumed to have the

$$\frac{1}{m^*(r)} = \frac{1}{m_0}(1 - \rho(r)) \quad (3)$$

where $\rho(r)$ is an isotropic function of the radial variable r that expresses the change of the projectile's mass with distance r from the center of the target nucleus [5]. The notion of an effective mass has also been introduced in relation to relativistic optical models that give rise to significantly reduced effective masses [16]. For example, the relativistic effects on the nucleon mean free path in the intermediate energy range were investigated in Ref. [17]. In addition, the idea of an effective mass was considered for the specification of a microscopic optical model potential Ref. [8]. At this point we stress that the gradient term in our model is different from the Darwin term present in the nonrelativistic equivalent of the Dirac-based relativistic formulation [18] because, firstly, our effective mass is real while the Darwin term is complex. Secondly, in the nonrelativistic equivalent of the Dirac formulation, the Darwin term is closely coupled to the spin orbit potential while our gradient term is independent of the spin orbit term [5]. Furthermore, unlike the case for the relativistic models mentioned above, our gradient term is expressed as an effective mass that has a radial dependence, rather than in terms of an additional scalar potential that is added to the mass. Finally, the effective mass formulation has also been introduced in the field of condensed matter physics calculations [19, 20, 21].

Other phenomenological forms of nonlocalities have also been considered such as a parity-dependent term in the optical potential [22, 23] for low energy scattering of protons from ^{16}O . This parity dependence was later justified tentatively in terms of the Feshbach channel coupling effect [13, 24]. A presently much used nonlocality, first introduced by Frahn and Lemmer [25] and further developed by Perey and Buck in the 1960ies, is given in the form of an integral kernel $K(r, r')$ that acts on the wave function. Perey and Buck also obtained a local equivalent potential in the corresponding Schrödinger equation [26].

A non local potential $V(r, r')$ acts on the nonlocal wave function ψ_{NL} in the form $\int V(r, r')\psi_{NL}(r')dr'$, while a local potential $V(r)$ acts as a factor $V(r)\psi_L(r)$, where $\psi_L(r)$ is a local wave function. Like the local wave function, ψ_{NL} is a function of one radial coordinate only. In the nuclear interior, a local potential leads to local wave functions which are different from the corresponding nonlocal ones obtained using a nonlocal potential, even though

both forms may lead to the same asymptotic phase shift. This is known as the Perey effect [6], and may be expressed quantitatively as

$$\psi_{NL}(r) = F(r)\psi_L(r), \quad (4)$$

where $\psi_L(r)$ and $\psi_{NL}(r)$ are the local and nonlocal wave functions respectively, while the Perey damping factor is defined as $F(r)^2$ [27]. In the nuclear interior, an attractive nonlocal potential results in a reduction in the nonlocal wave function ($F(r) < 1$), while for a repulsive potential the situation is reversed leading to an enhancement of $\psi_{NL}(r)$, that is, $F(r) > 1$. As discussed by Austern [6], nonlocality arises when one singles out from the overall wave function that part which describes the motion in the elastic channel. Outside the nucleus $\psi_{NL}(r)$ is the full physical wave function of the system. However, inside the nucleus part of the wave function of the system is hidden in the invisible (inelastic) channels, which results in a suppression of the nonlocal elastic wave function compared to the local one. Conversely, for a repulsive nonlocal potential, the particles of the target nucleus are driven away from the incident particle. However, conservation of the flux demands that the incident particle carries more flux in order to compensate for the part of the channel space which has been suppressed [6]. A more recent work employed nonlocal potentials in three-body direct nuclear reaction calculations and an important nonlocality effect in transfer reactions was reported [28].

In this work our aim is to show that the presence of a velocity-dependent part in the conventional, phenomenological optical potential simulates a source of nonlocality. This is achieved by showing that, in the interior of the nucleus, the nonlocal wave functions are different from the corresponding local ones obtained in the absence of a gradient term ($\rho(r) = 0$) in accordance with the Perey effect. We shall also demonstrate that this effect is angular momentum dependent.

2 Nonlocality due to the gradient term

In Ref. [5], we considered a novel velocity-dependent optical potential of the form given in Eq. (2) based on the change of mass of the incident nucleon, and obtained excellent fits for the elastic $N - {}^{12}C$ angular distribution data in the low energy range of 12 – 20 MeV. Most notably the model reproduced well the prominent large-angle, backscattering minima which depend sensitively on the incident energy, and which are a sign of the presence of nonlocalities

[13, 14]. The local potential consisted of central and spin orbit potentials and was taken to have the form:

$$V(r) = -V_0 f(r, x_0) + 4ia_w W \frac{df(r, x_w)}{dr} + 20 \left(\frac{\hbar}{\mu c} \right)^2 (V_{so} + iW_{so}) \frac{1}{r} \frac{df(r, x_{so})}{dr} \vec{\sigma} \cdot \vec{I}, \quad (5)$$

where x_0 stands for (r_0, a_0) and so on for the rest of the terms. In addition, the function $f(r, r_j, a_j)$ has the following Woods-Saxon form:

$$f(r, r_j, a_j) = \frac{1}{1 + \exp[(r - r_j A^{1/3})/a_j]}, \quad (6)$$

with A being the mass number of the target nucleus. In order to reduce the number of fit parameters (12) we used the same set of parameters for the real and imaginary parts of the spin-orbit term. In what follows the nonlocal wave function $\psi_{NL}(r)$ is the solution of the Schrödinger equation corresponding to the velocity-dependent potential $V(r, p)$ given in Eq. (2). The potential parameters are adjusted to obtain a best fit to the experimental $N - {}^{12}C$ elastic scattering angular distribution. The local wave function $\psi_L(r)$, however, is the solution of the Schrödinger equation corresponding to the equivalent local potential $V(r)$ whose parameters are adjusted to (i) fit experimental data and (ii) produce the same phase shift values as those obtained using the nonlocal potential $V(r, p)$. In the local equivalent potential case, the effect of nonlocalities in the scattering process are manifested as a variation of the potential parameters with energy. This procedure is not unique, but is guided by the conventionally used method to fit angular distributions. The local wave function obtained in this way is not equal to the trivially equivalent local wave function, which is obtained by starting the fit in the presence of a velocity dependent term, and then mathematically transforming the $d \psi_{NL}(r)/dr$ away by a renormalization of the wave function $(1 - \rho(r))^{1/2} \psi_{NL}(r)$ [29].

In Sec. 3, we demonstrate that the nonlocal wave function $\psi_{NL}(r)$ is indeed different from the local wave function $\psi_L(r)$ in the nuclear interior, which supports our hypothesis that the gradient term introduced into the conventional phenomenological optical potential simulates a source of non-locality. In addition, we find that the enhancement or suppression of the nonlocal wave function relative to the local one to be angular momentum dependent. This is in line with earlier works that modeled the NA elastic scattering by introducing a parity-dependent term $(-1)^l$ into the optical potential [23].

3 Results and Discussion

As described above, the parameters of the equivalent local potential $V(r)$ given in Eq. (5) were adjusted until (i) a best fit for the elastic $N - {}^{12}C$ angular distribution data was obtained and (ii) the resulting phase shifts were almost identical to the ones obtained corresponding to the nonlocal fits in Ref. [5]. The fits for the 12, 14 and 16 MeV incident nucleon energies are shown in Fig. 1, where the experimental data are taken from Ref. [30]. The equivalent local potential fit parameters as a function of the incident energy and orbital angular momentum are shown in Table I.

E_{lab}	V_0	r_0	a_0	W	r_w	a_w	$V_{so} + iW_{so}$	r_{so}	a_{so}
MeV	MeV	fm	fm	MeV	fm	fm	MeV	fm	fm
12	44.0	1.33	0.48	7.2	1.30	0.34	$17 + 4 i$	1.10	0.08
14	50.0	1.24	0.56	5.8	1.22	0.53	$12 + 0 i$	1.05	0.11
16	43.0	1.40	0.37	6.0	1.55	0.28	$26 + 8 i$	1.00	0.11
18	47.5	1.44	0.50	8.7	1.10	0.41	$20 + 15 i$	0.91	0.09
20	41.0	1.61	0.51	8.0	1.36	0.40	$20 + 5 i$	0.80	0.15

Table 1: Conventional optical model fit parameters for the $N - {}^{12}C$ elastic scattering angular distribution. The potential terms are given by Eq. (5).

Clearly, the overall behavior of the angular distributions has been reproduced. However, the fine details of the differential cross sections are less well described compared to the corresponding fits obtained using the phenomenological velocity-dependent optical potential [5]. The large angle back scattering minima (associated with nonlocalities) at 18 and 20 MeV were poorly reproduced by the local optical potential and hence are not shown.

Evidently, the fit parameters are energy sensitive, which indicates the presence of sources of nonlocalities. Most notable among them being the variation in the strength of the central potential V_0 . The corresponding non-local (velocity-dependent) optical potential fit parameters (given in Tables I and II of Ref. [5]) are less energy dependent indicating that the introduced gradient term only accounts for part of the nonlocalities present in the optical model. In a future work, we intend to investigate other sources of nonlocalities in the phenomenological optical potential.

In this work we have also investigated the suppression and enhancement effects of the nonlocal wave function $\psi_{NL}(r)$ as a function of the incident nucleon energy in addition to the orbital angular momentum. The Perey factor $F(r)$ defined by Eq. (4) has been determined by comparing the maxima of the real parts of $\psi_{NL}(r)$ and $\psi_L(r)$ in the nuclear interior. Since the gradient term simulating the nonlocality is purely real, the imaginary part of $\psi_{NL}(r)$ is almost identical to the corresponding local one. Further, the local wave function has been normalized such that it coincides with the nonlocal one in the asymptotic region. The damping (or enhancement) factor $F(r)^2$ as a function of the incident energy as well as the orbital angular momentum is shown in Table 2.

E_{lab}	angular momentum quantum number l					
MeV	0	1	2	3	4	5
12	1.746	0.838	0.876	1.451	1.00	1.00
14	1.44	0.779	0.835	1.477	1.00	1.00
16	1.134	0.882	1.171	0.918	1.00	1.00
18	1.355	0.636	1.227	2.001	1.00	1.00
20	0.928	0.700	0.988	1.522	0.895	1.00

Table 2: Perey factor $F(r)^2$ as defined in Eq. (4). The values are calculated at the peaks of the nonlocal and local wave functions in the nuclear interior.

By inspecting Table 2 it is evident that the suppression and enhancement effects are both energy and angular momentum dependent. Apart from the 20 MeV incident energy case, where the angular distribution fits are poorer than those obtained at lower energies, the s -wave nonlocal wave function is enhanced. For p -wave scattering a suppression is noted for all incident energies. Corresponding to $l = 2$, however, we have a mixed situation where the nonlocal wave function is suppressed for low energies and enhanced corresponding to the higher 16 and 18 MeV ones. However, the 20 MeV case stands out again and results in a small suppression. For the $l = 3$ case, we have an enhancement at all energies apart from the 16 MeV incident energy. For $l \geq 4$, the nonlocal wave function is too small in the interior region of the nucleus to feel the effect of the gradient term, which simulates the nonlocal-

ity, hence $F(r)^2$ reduces to close to unity. As an illustration, in Figs. 2 and 3 we show the real parts of the local $\psi_L(r)$ and nonlocal $\psi_{NL}(r)$ wave functions corresponding to different angular momentum quantum numbers all at the same 14 MeV incident neutron energy. As shown in Fig. 2 (a), for the s -wave case the nonlocal wave function (solid line) is clearly enhanced in the nuclear interior compared to the local one (dotted line). Further, Figs. 2 (b) and 2 (c) corresponding to $l = 1$ and $l = 2$ respectively show a suppression rather than an enhancement. For $l = 3$, the maximum of $\psi_{NL}(r)$ coincides with the peak of the gradient term (dash-dotted line), which explains the distortion of $\psi_{NL}(r)$ inside the nucleus as shown in Fig. 3 (a). Finally, for the $l = 4$ case, the nonlocal wave function in the nuclear interior is too small to be affected by the gradient term that simulates the nonlocal effect and acts in the internal nuclear region. Therefore, the two wave functions coincide in the interior region as can be seen in Fig. 3 (b). The fact that $\psi_L(r)$ and $\psi_{NL}(r)$ coincide in the external region (for all angular momenta) is guaranteed by the adopted normalization method explained above. In addition, since the nonlocality is simulated in terms of a purely real term only the real part of the nonlocal wave function is affected, the imaginary part is almost identical with the corresponding local one and hence is not shown in the figures.

4 Trivially local equivalent potential

The Schrödinger equation corresponding to a velocity-dependent potential $V(r, p)$ can be transformed into an equation for an equivalent local but energy-dependent one, $U(r, E)$, for a trivially equivalent local wave function $\chi_L(r)$ through the following transformation on the nonlocal wave function $\psi_{NL}(r)$ [29];

$$\psi_{NL}(r) = \frac{\chi_L(r)}{\sqrt{1 - \rho(r)}}, \quad (7)$$

which, as derived in [31], leads to the following energy-dependent term in the equivalent local potential;

$$\frac{\rho(r)k^2}{1 - \rho(r)}, \quad (8)$$

where, as defined in Ref. [5], ρ is given by

$$\rho(r) = \rho_0 a_\rho d/dr \left\{ \frac{1}{1 + \exp[(r - r_\rho A^{1/3})/a_\rho]} \right\}, \quad (9)$$

In this case the values of the fitting parameters for the nonlocal potential $\hat{V}(r, p)$, given by Eq. (2), and the local but energy dependent one $U(r, E)$ are identical. Consequently, $\psi_{NL}(r)$ and $\chi_L(r)$ share the same values of the scattering phase shifts. The ratio of the nonlocal wave function $\psi_{NL}(r)$ to the local equivalent one is, by Eq. (7), given by the expression

$$F(r) = \frac{1}{\sqrt{1 - \rho(r)}}. \quad (10)$$

This ratio is energy dependent, but has no variation with the orbital angular momentum. The energy dependence of $F(r)^2$ arises from the variation of ρ_0 , r_ρ and a_ρ with the incident energy. Table 3 shows the maximum values of $F(r)^2$ calculated as a function of the incident energy. The enhancement or suppression effects of the nonlocal wave function depend on the value of $\rho(r)$ as given by Eq. (9). Table 3 shows that $\psi_{NL}(r)$ is enhanced for all incident energies since $0 \leq \rho(r) < 1.0$.

E_{lab} (MeV)	$F(r)^2$
12	1.82
14	1.73
16	1.57
18	2.25
20	2.00

Table 3: Perey factor $F(r)^2$ as defined in Eq. (10). The table displays the maximum values of $F(r)^2$ as a function of the incident energy.

5 Conclusions

In a recent work we introduced a velocity-dependent term into the phenomenological, conventional optical potential. This resulted in excellent fits

for the $N - {}^{12}C$ elastic angular distributions even at large back scattering angles [5], which is a region known to be associated with nonlocalities [13, 14]. In the present study it has been shown that the velocity-dependent term simulates a source of nonlocality by showing that inside the target nucleus the nonlocal wave function $\psi_{NL}(r)$ is enhanced or suppressed compared to the corresponding local one $\psi_L(r)$ obtained in the absence of the velocity-dependent term. This is in accordance with the Perey effect [26]. In Table 1 we show the fit parameters for the equivalent local potential $V(r)$ given in Eq. (5) for the $N - {}^{12}C$ elastic scattering process. Clearly, the parameters are energy sensitive, and the strength of the local central potential V_0 shows the largest variation with incident energy. This indicates the presence of nonlocal effects in the scattering process. In this work and in Ref. [5] we proposed that the gradient term simulates a nonlocality, which is due to a change in mass of the incident nucleon when moving inside the nuclear matter of the target nucleus. The corresponding fit parameters for the nonlocal, velocity-dependent potential $V(r, p)$ (given in Tables I and II of reference [5]) show less variation with incident energy. This clearly indicates that there are other sources of nonlocalities like, for example, Pauli exchange and knock out processes in addition to the detailed structure of the target nucleus that still need to be accounted for. The local potential angular distribution fits are given in Figure 1, which show that the details of the angular distributions are less well described compared to the corresponding ones obtained in Ref. [5].

We have also shown that the enhancement or suppression effects are angular momentum dependent. In Table 2 we show the Perey factor $F(r)^2$ as a function of the incident energy as well as the orbital angular momentum quantum number l . The angular momentum dependence of $F(r)^2$ is suggestive of the inclusion of a parity dependent term $(-1)^l$ in the conventional optical potential [13, 24]. Since the gradient term simulating the nonlocality is purely real only the real part of $\psi_{NL}(r)$ is affected and the imaginary part is almost identical with the corresponding one for the local wave function. Figs. 2 and 3 show the real parts of local (dotted) and nonlocal (solid) wave functions for different angular momentum quantum numbers l all at the same incident energy of 14 MeV. Clearly, $F(r)^2$ is a function of orbital angular momentum.

Finally, we considered two methods to determine the local wave function $\psi_L(r)$ corresponding to the nonlocal one, $\psi_{NL}(r)$, that was obtained in Ref. [5]. The first corresponds to obtaining the best fit parameters for an

equivalent local potential such that $\psi_L(r)$ and $\psi_{NL}(r)$ are phase equivalent. The other obtains a trivially equivalent local wave function, which starts by fitting the data in the presence of the velocity-dependent term and then mathematically transforming the $d \psi_{NL}(r)/dr$ away by a renormalization of the wave function as given by Eq. (7). In this case the nonlocal $\psi_{NL}(r)$ and trivially local $\chi_L(r)$ wave functions share the same fit parameters and, by Eq. (10), $F(r)^2$ has energy dependence only as shown in Table 3. Since the values of $\rho(r)$, which is given by Eq. (9), range from zero to less than unity $\psi_{NL}(r)$ is enhanced for all energies.

In conclusion, the non local nature of the derivative term is justified here in terms of the properties of the corresponding Perey Damping Factor, which differs from unity by 20 to 30%. Furthermore, in future works we shall test the applicability of the introduced velocity-dependent potential to other systems like $p - {}^{16}O$ and $n - {}^{40}Ca$, in order to determine whether the surface term for $\rho(r)$ as defined in Eq. (9) is still valid for heavier nuclei, or whether it should be replaced by a volume term that is proportional to the nuclear density itself, rather than its gradient.

References

- [1] H. Feshbach, *Ann. of Phys.* **5** (1958) 357.
- [2] H. Feshbach, *Theoretical nuclear physics:Nuclear reactions* John Wiley and Sons, Inc. (1992)
- [3] P. Fraser, K. Amos, S. Karatagliidis, L. Canton, G. Pisent and J. P. Svenne *Eur. Phys. J. A* **35**, (2008) 69.
- [4] A.J. Koning and J.P. Delaroche, *Nucl. Phys. A* **713** (2003) 231.
- [5] M. I. Jaghoub, M. F. Hassan and G. H. Rawitscher, *Phys. Rev. C* **84** (2011) 034618.
- [6] N. Austern, *Phys. Rev.* **137** (1965) 752.
- [7] P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, Springer-Verlag, 1st edition, 1980, p. 212
- [8] K. Amos, P. J. Dortmans, H.V. von Geramb, S. Karatagliidis, and J. Raynal , *Adv. Nucl. Phys.* **25** (2000) 275.
- [9] L. Canton, G. Pisent, J. P. Svenne, D. van der Knijff, K. Amos, S. Karatagliidis, *Phys. Rev. Lett.* **94** (2005) 122503.
- [10] E. Cereda, M. Pignanelli, S. Micheletti, H. V. von Geramb, M. N. Harakeh, R. De Leo, G. D'Erasco, and A. Pantaleo *Phys. Rev.C* **26** (1982) 1941.
- [11] George H. Rawitscher, Nuclear Physics A475 (1987) 519.
- [12] K. Amos, L. Canton, G. Pisent, J. P. Svenne, D. van der Knijff, , *Nucl. Phys. A* **728** (2003) 65.
- [13] G. H. Rawitscher and D. Lukaszek, *Phys. Rev.C* **69** (2004) 044608.
- [14] E. D. Cooper, S. Hama, B. C. Clark and R. L. Mercer, *Phys. Rev. C* **47** (1993) 297.
- [15] L.S. Kisslinger, *Phys. Rev.* **98** (1955) 761.

- [16] B. D. Serot and J. D. Walecka, Advances in Nuclear Physics, edited by J. W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16.
- [17] T. Cheon, Phys. Rev. C **38** (1988) 1516.
- [18] L. G. Arnold and B. C. Clark, Phys. Lett. B **84** (1979) 46 .
- [19] M. R. Geller, and W. Kohn, *Phys. Rev. Lett.* **70** (1993) 3103.
- [20] L. I. Serra and E. Lipparini, *Euro. Phys. Lett.* **40** (1997) 667.
- [21] I. Gómez, E. Domínguez-Adame, and P. Orellana, *Physica E* (Amsterdam) **18** (2003) 372.
- [22] S. G. Cooper and R. S. Mackintosh, *Phys. Rev. C* **54** (1996) 3133.
- [23] S. G. Cooper, *Nucl. Phys. A* **618** (1997) 87.
- [24] G. H. Rawitscher, D. Lukaszek, R. S. Mackintosh and S. G. Cooper, *Phys. Rev. C* **49** (1994) 1621.
- [25] W. E. Frahn and R. H. Lemmer, *Nuovo Cim.* **5** (1957) 523.
- [26] F. G. Perey and B. Buck, *Nucl. Phys. A* **32** (1962) 353.
- [27] A. D. Mackellar and R. E. Schenter *Nucl. Phys. A* **178** (1971) 249.
- [28] A. Deltuva, *Phys. Rev. C* **79** (2009) 021602(R).
- [29] M. Krell, T. E. O Ericson, *Nucl. Phys. Phys. B* **11** (1969) 521.
- [30] Evaluated Nuclear Data File, Library: CENDL-3.1, Sub-Library: NSUB=10 (N), Authors: J. S. Zhang, Y. L. Han, X. J. Sun, H. C. Wu, File: MF4 Angular distributions of secondary particles, Section MT2 (N,EL)L0 Elastic scattering cross section for incident particles
- [31] M. Al Jaghoub, *Eur. Phys. J.A* **11** (2001) 175.

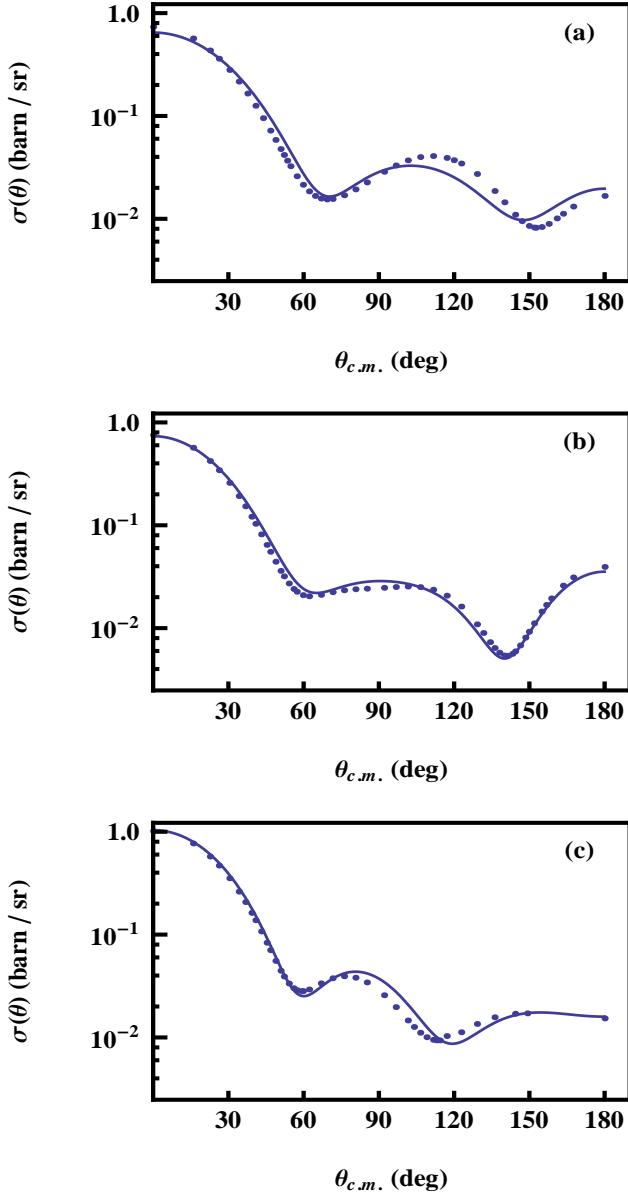


Figure 1: The conventional optical potential fits for $N - {}^{12}C$ elastic scattering at 12 (a), 14 (b) and 16 (c) all in units of MeV. The model parameters are given in Table 1. The data is obtained from reference [30].

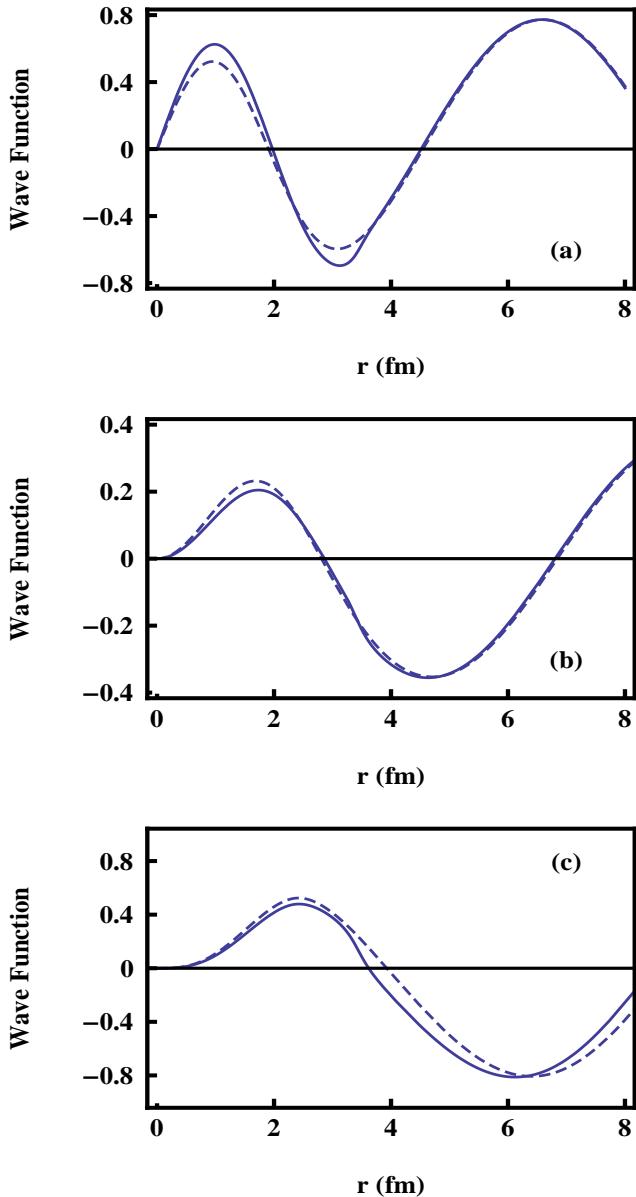


Figure 2: The real parts of the local (dotted) and nonlocal (solid) wave functions corresponding to (a) $l = 0$, (b) $l = 1$ and (c) $l = 2$ angular momentum quantum numbers. The local and nonlocal wave functions correspond to the local and nonlocal potentials given by Eqs. (2) and (5). All figures correspond to $E = 14$ MeV incident neutron energy.

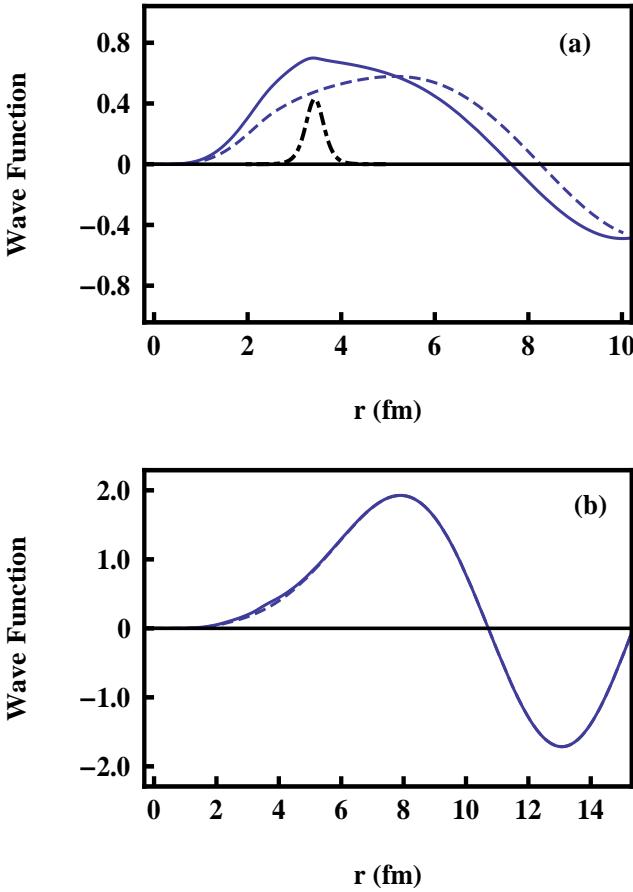


Figure 3: The real parts of the local (dotted) and nonlocal (solid) wave functions corresponding to the following angular momentum quantum numbers: (a) $l = 3$, and (b) $l = 4$. In (a) the dash-dotted peak corresponds to $\rho(r)$ defined in Eq. (9). The peaks of $\rho(r)$ and the nonlocal wave function almost coincide, which explains the distorted peak of the latter. The local and nonlocal wave functions correspond to the local and nonlocal potentials given by Eqs. (2) and (5). Both figures correspond to $E = 14$ MeV incident neutron energy.